Chapter 10 Circles

Circle: A circle is a collection of all points in a plane which are at a constant distance from a fixed point.

Centre: The fixed point is called the centre.

Radius: The constant distance from the centre is called the radius.

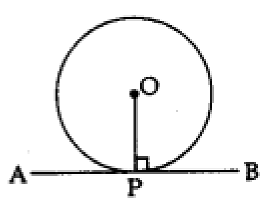
Chord: A line segment joining any two points on a circle is called a chord.

Diameter: A chord passing through the centre of the circle is called diameter. It is the longest chord.

Tangent: When a line meets the circle at one point or two coincidings The line is known as points, a tangent.

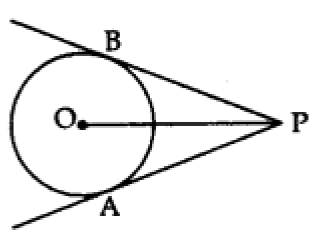
The tangent to a circle is perpendicular to the radius through the point of contact.

$\Rightarrow OP \perp AB$



The lengths of the two tangents from an external point to a circle are equal.

$\Rightarrow AP = PB$



Length of Tangent Segment

PB and PA are normally called the lengths of tangents from outside point P.

Properties of Tangent to Circle

Theorem 1: Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

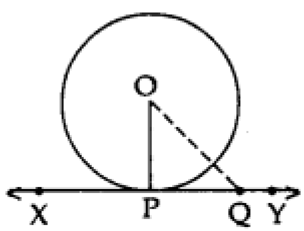
Given: XY is a tangent at point P to the circle with centre O.

To prove: OP ⊥ XY

Construction: Take a point Q on XY other than P and join OQ

Proof: If point Q lies inside the circle, then XY will become a secant and not a tangent to the circle

OQ > OP



This happens with every point on the line XY except the point P. OP is the shortest of all the distances of the point O to the points of XY

OP ⊥ XY ...[Shortest side is the perpendicular]

Theorem 2: A line drawn through the end point of a radius and perpendicular to it, is the tangent to the circle.

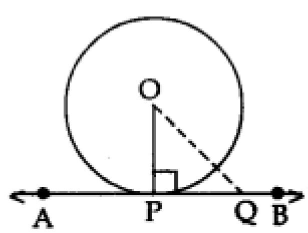
Given: A circle C(O, r) and a line APB is perpendicular to OP, where OP is the radius.

To prove: AB is tangent at P.

Construction: Take a point Q on the line AB, different from P and join OQ.

Proof: Since OP ⊥ AB

OP < OQ ⇒ OQ > OP



The point Q lies outside the circle.

Therefore, every point on AB, other than P, lies outside the circle.

This shows that AB meets the circle at point P.

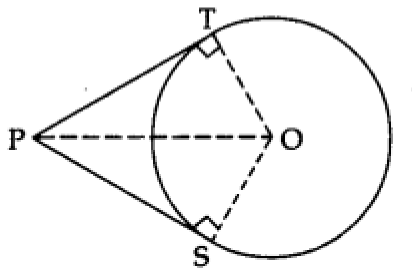
Hence, AP is a tangent to the circle at P.

Theorem 3: Prove that the lengths of tangents drawn from an external point to a circle are equal

Given: PT and PS are tangents from an external point P to the circle with centre O.

To prove: PT = PS

Construction: Join O to P, T and S.



Proof: In ΔOTP and ΔOSP.

OT = OS ...[radii of the same circle]

OP = OP ...[common]

∠OTP = ∠OSP ...[each 90°]

ΔOTP = ΔOSP ...[R.H.S.]

PT = PS ...[c.p.c.t.]

Note: If two tangents are drawn to a circle from an external point, then:

- They subtend equal angles at the centre i.e., ∠1 = ∠2.
- They are equally inclined to the segment joining the centre to that point i.e., ∠3 = ∠4.

  ∠OAP = ∠OAQ